M.Sc. course „Computational Engineering"

Summer 2001

Advanced Finite Element Methods

Written Examination on 6.8.2001

Last name:________________________ First name:______________________
(please write legibly)

exercise no.	1	2	3	4	sum
max. points	40	47	47	46	180
obtained points					

Important instructions

- Duration: 3 hours,
  first 40 minutes without,
  2 hours and 20 minutes with appliance.
- Proof the completeness of the exercises.
- Write your name onto all pages of the test.
- Write the solutions of part 1 of the exam directly onto the handout.
  Don't use your own paper.
- Hand in all pages of the examination.
- The solutions should include all auxiliary calculations.
- Programmable pocket calculators are only allowed without programs.
- The use of laptops or notebooks is not allowed.

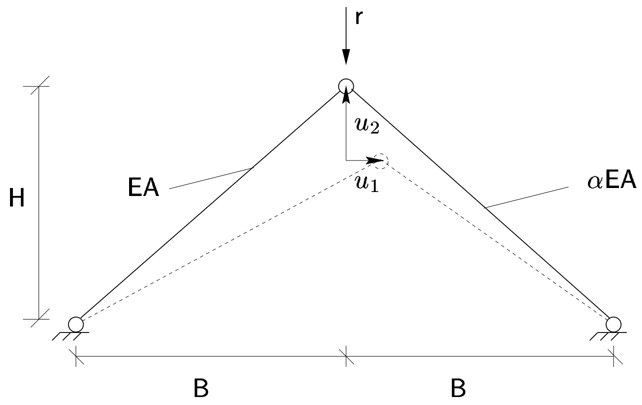
Exercise: 2 max. $\sum$ points: 47 obtained. $\sum$ points:

The equilibrium condition of the shown system can be formulated by the following equations:

$$\begin{aligned} \boldsymbol{r}_i(\boldsymbol{u}) &= \boldsymbol{r} \\ \boldsymbol{r}_i(\boldsymbol{u}) &= \frac{EA}{2L^3}\left\{(\boldsymbol{X}+\boldsymbol{u})\left[(2\boldsymbol{X}+\boldsymbol{u})\cdot\boldsymbol{u}\right]+\alpha\left(\bar{\boldsymbol{X}}+\boldsymbol{u}\right)\left[(2\bar{\boldsymbol{X}}+\boldsymbol{u})\cdot\boldsymbol{u}\right]\right\}, \end{aligned} \tag{1}$$

with

$$\boldsymbol{X}=\begin{bmatrix} B \\ H \end{bmatrix}, \bar{\boldsymbol{X}}=\begin{bmatrix} -B \\ H \end{bmatrix}, \boldsymbol{u}=\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \boldsymbol{r}=\begin{bmatrix} 0 \\ -r \end{bmatrix}. \tag{2}$$



H	=	0.65 [m]
B	=	1.0 [m]
EA	=	$2.1\cdot10^{-2}$ [KN]

Figure 1: Geometry and FE-model of the structure

a) Proof the following identity by means of the GATAUX derivative:

$$\frac{\partial}{\partial\boldsymbol{u}}\left\{(\boldsymbol{X}+\boldsymbol{u})\left[(2\boldsymbol{X}+\boldsymbol{u})\cdot\boldsymbol{u}\right]\right\}=\left[(2\boldsymbol{X}+\boldsymbol{u})\cdot\boldsymbol{u}\right]\mathbf{1}+2(\boldsymbol{X}+\boldsymbol{u})\otimes(\boldsymbol{X}+\boldsymbol{u}) \tag{3}$$

with

$$\mathbf{1}=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{4}$$

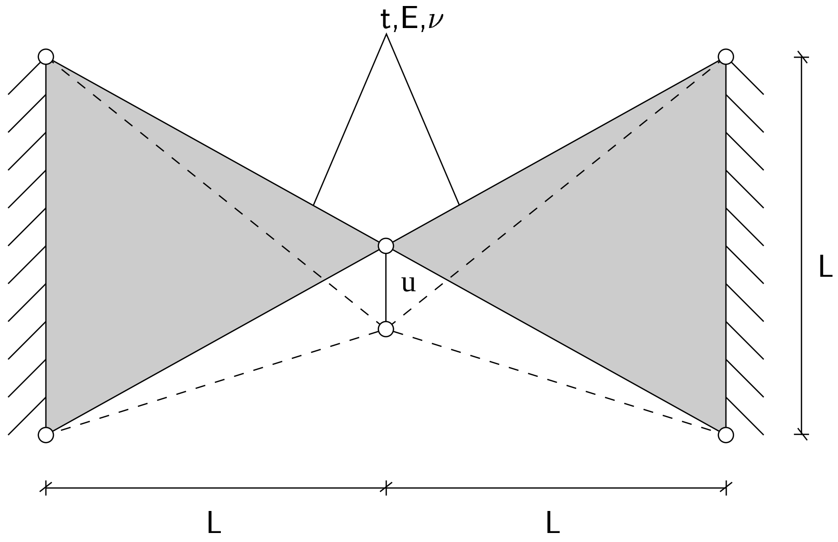
b) Compute the stiffness matrix $\boldsymbol{K}_T(\boldsymbol{u})$ for the nonsymmetric structure ($\alpha = 1.5$).

c) Derive the equation of the NEWTON-RAPHSON iteration scheme by linearizing the weak form of equilibrium.

d) Compute the deformation of the nonsymmetric system with a point load of $r = 1.2996 \cdot 10^{-3}$ using 2 iterations.

e) Chose a convergence criterion and check the convergence.

Exercise: 3 max. $\sum$ points: 47 obtained. $\sum$ points:



E	=2.1 [KN]
ν	=0.0
t	=0.1 [m]
L	=1.0 [m]
u	=0.2 [m]

Figure 2: Geometry and FE-model of the structure

Compute the response of the 2-D structure illustrated in the figure. The material behavior can be described by a scalar damage model. The elastic domain is approximated by the standard HOOKE's law. A hyperbolic equation for the relation between the damage and the internal variable

$$d(\kappa) = 1 - \frac{0.1}{\kappa} \tag{5}$$

is investigated for the modeling of the post peak response. With the definition of an equivalent strain measure

$$\eta(\boldsymbol{\varepsilon}) = \sqrt{\boldsymbol{\varepsilon}^T \boldsymbol{\varepsilon}}, \tag{6}$$

the model is completed.

a) Compute the approximated strains $\boldsymbol{\varepsilon}^T = [\varepsilon_{11},\ \varepsilon_{22},\ 2\varepsilon_{12}]$ using a 3 node plane stress triangular element (CST).

b) Compute the current damage variable d.

c) Compute the components $\boldsymbol{\sigma}^T = [\sigma_{11},\ \sigma_{22},\ \sigma_{12}]$ of the stress tensor.

d) Proof the following lemma for CST elements and a scalar damage model ($\boldsymbol{\sigma} = (1-d)\ \boldsymbol{C}\ \boldsymbol{\varepsilon}$):

$$\boldsymbol{K}_d^T = (1-d)\ \boldsymbol{K}_{el}^T. \tag{7}$$

In Equation (7) $\boldsymbol{K}_{el}^T$ is the element stiffness matrix of the undamaged structure, whereas $\boldsymbol{K}_d^T$ takes stiffness degradation within the element into account.

Exercise: 4 max. $\sum$ points: 46 obtained. $\sum$ points:

The geometrically nonlinear analysis of a slab has resulted in the deformed shape in Fig. 3. The computation has been carried out by means of a four node finite element with bilinear displacement interpolations.

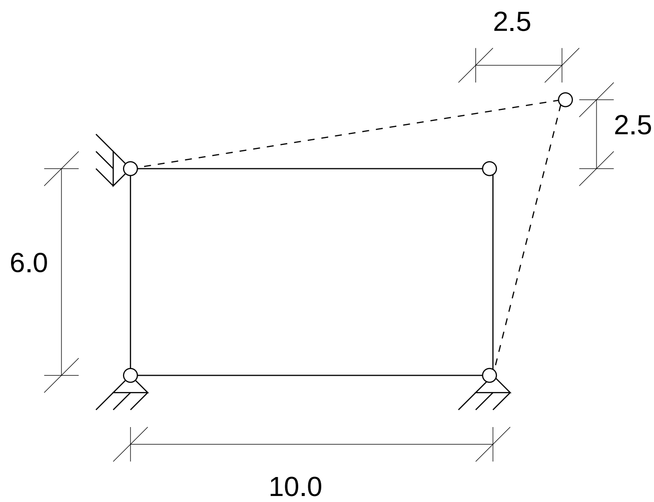


Figure 3: Reference and deformed configuration

The following aspects have to be addressed:

a) Compute the JACOBI matrix and determinant.

b) Compute the displacement field with respect to physical coordinates.

c) Compute the deformation gradient.

d) Compute the distribution of the density of the deformed configuration with respect to the density ρ_0 of the reference configuration.

e) Compute the components E_{11}, E_{22}, E_{12} and E_{21} of the GREEN-LAGRANGE strain tensor.